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Logic, Fast and Slow: The Persistent Difficulty of the Monty Hall Problem

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ABSTRACT

Evolutionary cognitive psychology is equipped to answer questions regarding not only human reasoning but also its limitations. Given that the field argues for naturally selected reasoning capacities (either broad or modular), the causes of certain cognitive errors and biases are of important interest. Kahneman (2011) has investigated and explicated the many fallacies in human logic that can lead people to make less than optimal judgments and decisions. Evolutionary cognitive psychologists have examined both probability judgments and conditional reasoning. Taken together, it would appear that evolutionary psychology could shed light on the notion that humans think both 'fast and slow.' This study examined two aspects of logical problem-solving hypothesized to be necessary for deducing the optimal response to the Monty Hall problem. The authors investigated the effects of a demonstration designed to emphasize the logic of the Monty Hall problem and to facilitate perspective-change, and the authors investigated the effects of counterfactual reasoning tasks. Alone, these two conditions—the demonstration and counterfactual reasoning—did not improve performance over controls. When combined, they did significantly improve subjects' performance. We argue that subjects' strong tendency to respond illogically to the Monty Hall problem is an example of fast, System 1 thinking and that the combined cognitive influences of a logical demonstration and counterfactual reasoning facilitated slow, System 2 thinking. Further we argue that slow, System 2 thinking operates with two subsystems called 'fast logic' and 'slow logic.'

KEYWORDS

Problem-Solving, Probabilistic Reasoning, Logic

The aim of the present study was to further examine the enduring difficulty people have in grasping the logic behind the optimal response to what is known as

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the Monty Hall problem[1]. In this scenario (named after the game show host of the television show "Let's Make a Deal"), there are three doors from which a contestant is asked to choose. The contestant is also told that there is a prize behind one door and a goat behind each of the other two. After the contestant selects a door, Monty Hall, knowing which of the doors has the prize and which have the goats, then opens one of the other two remaining doors to reveal one of the goats. It is crucial to note that Monty is *required* to show one of the two goats at this point, never revealing the prize. The contestant is then asked if he or she would like to *stay* with the initially selected door or *switch* to the one remaining door. Though it is counterintuitive to most, it is to the contestant's advantage to *switch* because there is 33 percent chance that the prize is behind the initially selected door, but there is a 67 percent chance that it is behind the other remaining door.

If this seems dubious, examine Figure 1 below. Displayed is one possible arrangement of the three doors (the prize and the two goats). In this particular, hypothetical set of scenarios, we will imagine what would happen if the *first* door is initially chosen (indicated by **bold** font) and with the decision to *stay* after Monty Hall has eliminated one of the goats as an option (indicated by a ~~strikethrough~~). Note the corresponding outcomes for each of the three scenarios.

Figure 1. *A set of scenarios based on the decision (1) to pick* door 1 *and (2) to* stay *with the corresponding outcomes.*

Door 1	Door 2	Door 3	Decision	Outcome
Prize	~~Goat~~	Goat	Stay	Win
Goat	Prize	~~Goat~~	Stay	Lose
Goat	~~Goat~~	Prize	Stay	Lose

It is clear by looking at the corresponding outcomes: by staying the contestant runs the risk of losing out on the prize two out of three times. Next, let's see what happens if the contestant had decided to *switch* in the same three identical scenarios, as shown in figure 2.

Figure 2. *A set of scenarios based on the decision (1) to choose* door 1 *and (2) to* switch *with the corresponding outcomes.*

Door 1	Door 2	Door 3	Decision	Outcome
Prize	~~Goat~~	Goat	Switch	Lose
Goat	Prize	~~Goat~~	Switch	Win
Goat	~~Goat~~	Prize	Switch	Win

This time, by choosing to switch in all three scenarios, it is clear that the contestant has put herself in position to win the prize two out of three times. This

[1]It is also referred to as the 'Monty Hall dilemma,' 'Monty Hall puzzle,' and the 'Three Door problem.'

matrix can be repeated two more times, in which door 2 is chosen each time and door 3 is chosen each time, to reveal the statistical advantage of switching across all possible scenarios.

Previous research has shown that the vast majority of the time subjects choose to stay rather than switch, not realizing that the odds are much better that the prize is behind one of other remaining doors compared to the initially selected door. Often, subjects believe that the remaining choice is 50 : 50 not 33 : 67 (Tor & Bazerman, 2003). Further, subjects who do actually switch rarely do so because they have grasped or intuited the logic (Franco-Watkins, Derks, & Dougherty, 2003). One of the first psychological studies on the Monty Hall problem found that only 13 percent of subjects switched because they understood the probabilistic advantage (Granberg & Brown, 1995), and this percentage is indicative of the success rates of other studies using the Monty Hall problem conducted since (Burns & Wieth, 2004).

Several studies have examined the role of repeated trials of the Monty Hall problem, attempting to reveal any learning of the strategy that might take place with those who do not apprehend the logic initially. Some of these studies show that regret and counterfactual thinking (after losing out because of switching) tend to reinforce the human bias to “go with my first choice” (see Gilovich, Medvec, & Chen, 1998; Kirkebøen, 2013; Murphy & Douma, 2000; Petrocelli & Harris, 2011). It has even been indicated that regret inhibits learning effects of repeated Monty Hall problem trials in so far as primates (presumably lacking in counterfactual regret) may out-perform humans and intuit the underlying probability more effectively than humans (Klein, Beran, Evans, & Barrett, 2008). At the very least, a later study found that the two species (monkeys and humans) learned to switch after repeated trials at equal rates, suggesting that something like counterfactual thinking inhibits humans from displaying their superior reasoning ability (Klein, Evans, Schultz, & Beran, 2013). It should also be noted that a few studies, with conflicting or inconsistent findings, have examined the extent to which pigeons, with behavioral reinforcement schedules, learn that the optimal strategy is to switch, and some of these have been comparative among humans, monkeys, and pigeons (see Herbranson, 2012; Herbranson & Schroeder, 2010; Mazur & Kahlbaugh, 2012; Stanger, Rayburn-Reeves, & Zentall, 2013).

Using only a single trial for each subject, another group of studies has examined the various cognitive skills, tasks, hints, demonstrations, etc. which may facilitate correctly reasoning the Monty Hall problem. For example, subjects with higher capacities for working memory were shown to outperform subjects with more limited working memories (De Neys & Verschueren, 2006). One study demonstrated that explaining the logical structure was of only limited assistance (Tubau, 2008). This study also concluded that demonstrating natural frequencies was helpful (e.g., “1 out of 100”), while cumulative frequencies were not (e.g., “0.1”). Further, the usefulness of the frequencies was contingent upon the subject’s preexisting math ability. Also, graphs of the probability of the Monty Hall problem were not helpful. Ultimately, it was concluded that a cognitive prerequisite for correctly reasoning the problem was “adequate numerical representation.”

Krauss and Wang (2003) demonstrated that various activities designed to encourage the logical thinking associated with grasping the logic of the Monty Hall problem could improve subjects’ performance. In their study, subjects who performed tasks designed to encourage a 'frequency-based' understanding as well as tasks which facilitated 'perspective-change' significantly outperformed subjects who performed tasks which facilitated what Johnson-Laird has called 'mental models.' Overall, these studies demonstrate just how difficult it is for subjects to apprehend the underlying logic so that the choice to switch is understood as optimal.

In our study, we investigated two novel approaches to facilitating correct responses to the Monty Hall problem. Firstly, we wanted to explore whether a demonstration amplifying the underlying logic would encourage and work in tandem with the kind 'perspective change' discussed by Krauss and Wang (2003), since their study had demonstrated its potential effectiveness. In the present study, we used a demonstration which we hoped would accomplish two goals: 1) interactively demonstrate and strongly emphasize the basic logic of the Monty Hall problem, and 2) place the subject in the omniscient position of Monty Hall (i.e., knowing the arrangement of the prize and the goats behind the three doors).

Secondly, we wanted to further explore the role of counterfactual reasoning. As cited above, several studies have examined how counterfactual reasoning seems to inhibit learning or intuiting the underlying probability of the Monty Hall problem. But this has been after multiple trials and after learning the outcomes of the decisions. Other studies have potentially indicated that counterfactual conditional reasoning may be of assistance when introduced before the presentation of the Monty Hall problem. For example, one study found that hints which emphasize that Monty Hall's actions (i.e., opening one of the doors to reveal a goat only) are *conditional* upon which door was chosen initially by the contestant increased subjects' likelihood of choosing to switch (Jiang & Quinglin, 2006). Tubau and Alonso (2003), in conducting a series of three experiments, concluded that correctly reasoning the Monty Hall problem is dependent upon the ability to 'consider different possibilities.' Another study directly tested the extent to which a counterfactual conditional reasoning question related to correctly conceptualizing the probability ('causal structure') inherent in the Monty Hall problem (which, following Rehder [2003], they term 'the collider principle') (Burns & Wieth, 2005). In two experiments, they found that for subjects who had already established some understanding of the causal structure, the counterfactual question encouraged them to switch for the correct reason.

Counterfactual conditionals contain within them underlying predicate logic in the subjunctive form of 'if *p* had happened, then *q* would have happened' (Brain & O'Brien, 1998). They can also be in the form of 'if *p* had not happened, then *q* would not have happened.' Altogether, there are four main logical types of counterfactuals: $p \rightarrow q$, $-p \rightarrow -q$, $-p \rightarrow q$, and $p \rightarrow -q$. It has been shown that exposure to counterfactuals can act as a type of prime (Kosegarten, 2010; Galinsky & Moskovitz, 2000; Galinsky, Moskowitz, & Skurnik, 2000). We reasoned that using counterfactuals as a prime might facilitate correct reasoning on the Monty Hall problem in five possible ways: 1) the logical structure may facilitate logical thinking in general; 2) they might encourage a turn away from the typical bias of 'going with one's first guess;' 3) they might encourage the consideration of different possibilities; 4) they might emphasize the 'causal structure' of the Monty Hall problem as discussed by Burns and Wieth (2005); and 5) they might serve to reinforce the logic displayed in the 28-door demonstration (mentioned above and discussed in Methods below).

Because the grasping of the underlying logic to the reasoning behind the decision to 'switch' is so difficult, we hypothesized that the demonstration would improve performance or understanding moderately above controls. We also hypothesized that reading a series of counterfactual conditional statements would improve performance or understanding moderately above controls. For both of these conditions, however, we believed that more subjects would switch than controls but that a good portion would not necessarily do so for the correct reason. Lastly, we hypothesized that the two tasks together (both the demonstration and reading counterfactual statements) would have an additive effect which would significantly improve understanding of the Monty Hall problem and that this would be reflected in the number of 'switches' for the correct reason.

METHODS

Subjects

One-hundred-two subjects (58 females, 44 males) were recruited from a Northeastern university in the United States. Ages ranged from 17- 20-years-old (*M* = 18.03*, SD* = .59). Three of the subjects were excluded from the study because they reported that they knew the Monty Hall problem and that they knew the reason to switch was because of the probabilistic advantage. Also, three other subjects were excluded because they failed to understand the demonstration. This is explained further in the Results section. The study will thus report on the remaining 96 subjects (54 females, 42 males).

Materials and Procedure

The study used a 2 X 2 factorial design (see Figure 3). The 96 subjects were randomized into four conditions: three experimental and one control. All subjects were eventually tested on their response to the Monty Hall problem (the DV) and to explain their reasoning.

Figure 3. *Design of the experiment outlining the four conditions.*

		Counterfactual Conditional Reasoning Tasks	
		Yes	No
Demonstration	Yes	*Demonstration & CCR*	*Demonstration Only*
	No	*CCR Only*	*Control Group*

Subjects assigned to the control condition were simply tested on their ability to correctly solve the Monty Hall problem and to explain their reasoning for which of the two options they chose ("stay" or "switch"). This method was similar to Krauss & Wang (2003). Please see Appendix A for the design of the paper-and-pencil version of the Monty Hall problem.

In the Demonstration Only condition, subjects individually watched a brief PowerPoint presentation on a computer screen of a demonstration of a Monty-Hall-type problem that was designed to strongly emphasize the logic behind the strategy of switching and to facilitate perspective-change in the form of imagining both what is like to be the contestant and Monty Hall (see Appendix C). The presentation depicted a Monty-Hall-problem-type scenario in which the subject was shown 28 doors arranged in a 7 X 4 grid (the equivalent of 'the doors' in a typical Monty Hall problem scenario). As the experimenter slowly clicked through the PowerPoint demonstration, he deliberately recited a script:

> *Imagine you're playing a game with your friend. Here are 28 doors. Behind 27 of these doors, there is no prize. However, behind this door* [indicated with an arrow on the screen—#13 were we to number the doors left-to-right] *is a prize—a car. You know where the prize is, but your friend has no idea at all. You ask your friend to randomly select a door he thinks the prize is behind, and he chooses this door* [indicated with an arrow on the screen—#17 were we to number the doors left-to-right]. *Next, you eliminate all of the doors except the one that has the prize and the one your friend chose. Now, your friend has a choice: He can either stay with the door he first chose when he knew nothing, or he can switch to the other remaining door now that 26 of the 28 doors have been eliminated? What should your friend do? Should he stay with the first door or switch to the other one?* [The experimenter paused for the subject to answer.] *Right! He should switch because the odds that the car is behind the 1st choice is 1/28 (4%). But the odds that it's behind the other remaining door are 27/28 (96%).*

Following this demonstration, the subjects were then given the paper-and-pencil version of the Monty Hall problem and were asked to complete it, filling out the section asking for a description of his or her reasoning.

In Counterfactual Conditional Reasoning (CCR) Only condition, the subjects performed a series of twenty counterfactual conditional reasoning tasks that were loosely based on statements used by Santamaria, Espino, and Byrne (2005). However, we made one major adjustment: we made the statements into 'tasks' by making the last word in each sentence a fill-in-the blank (see Appendix B). It was thought that this would make the priming experience stronger and more interactive. We also introduced some of our own counterfactual statements that differ from Santamaria, et. al. (2005). There were five representative sentences each for the four different types of counterfactuals: 'If *p* had happened, then *q* would have happened;' 'If *p* had *not* happened, then *q* would *not* have happened;' 'If *p* had *not* happened, then *q* would have happened;' and 'If *p* had happened, then *q* would *not* have happened.' The counterfactual conditional reasoning sentences were initially jumbled (so that the twenty sentences were in a random sequence), and they were presented to all subjects in that same, random order. Answers that were not exactly what we had in mind but were essentially the same were considered correct (e.g., "moist" instead of "wet," or "pointy" instead of "sharp"). Note that the word "not" is *italicized* each time it arises in a counterfactual conditional reasoning sentence for emphasis. Also, subjects were asked to read each sentence *very* carefully because they are similar to one another. After completing the CCR tasks, the subjects were asked to respond to the Monty Hall problem and to explain their reasoning

In the Demonstration + Counterfactual Conditional Reasoning (CCR) condition, subjects went through the 28-door demonstration identical to that in the Demonstration Only condition and then completed a series of counterfactual conditional reasoning tasks identical to those in the Counterfactual Only condition. The order of these tasks was fixed. Following the 28-door demonstration and CCR tasks, the subjects were asked to respond to the Monty Hall problem and to explain their reasoning.

RESULTS

Preliminary Analysis

As stated above, we eliminated three subjects because of their previous knowledge of the Monty Hall problem leaving 99 remaining. Three other subjects were eliminated from the analysis because at the end of the Demonstration phase, when asked whether their friend should 'stay' or 'switch,' they said that their friend should 'stay.' This demonstrated to us that there was clearly no effect of the demonstration on their thinking and which would not have any measurable or meaningful effect on the dependent variable. Ultimately, the analysis was on the remaining 96 subjects.

We also asked the subjects their 'familiarity' with the Monty Hall problem. Seventeen subjects reported that they felt they were somewhat familiar with the problem but were unaware of its solution, and 79 subjects reported no familiarity at all. There were no statistically salient differences between these two groups.

Four of the eliminated subjects had been randomly assigned to the control condition and two to the Demonstration Only condition, thus the number of subjects assigned to each cell was somewhat uneven. One more note: not all subjects assigned to either the Counterfactual Conditional Reasoning condition or the Demonstration + Counterfactual Conditional Reasoning condition (n = 56) correctly completed all of the counterfactual conditional reasoning tasks. However, the vast majority of subjects did correctly complete all twenty (n = 48). A few answered 19 items correctly (n = 3); a few answered 18 items correctly (n = 3); and two subjects answered 17 items correctly. In general the errors occurred on the 'double-negative' counterfactuals (e.g., If it had *not* been raining, then the grass would *not* be _____ [wet]"), and otherwise errors seemed not to follow a pattern. In terms performance on the Monty Hall problem there were no statistically significant differences among these subjects.

Main Analysis

As predicted the overall frequency with which subjects chose to switch was low. Sixty-nine subjects (72%) chose to *stay* with the first selected door, and 27 subjects (28%) chose to *switch* to the other door. As shown in Tables 1 and 2, of the 27 subjects who switched, 15 (56%) were in the Demonstration + CCR Task condition.

Table 1. *Frequency of the decisions to 'stay' or 'switch' by condition with percentages within each condition.*

	Decision				
Condition	Stay		Switch		Total
Control	15	79%	4	21%	19
Demonstration Only	18	86%	3	14%	21
CCR Tasks Only	19	73%	5	27%	26
Demo + CCR Tasks	15	50%	15	50%	30
Total	69	72%	27	28%	96

Table 2. *Cumulative frequency of the decision to 'switch' across the four conditions.*

Condition	**Frequency**	**Relative Frequency**	**Cumulative Frequency**
Control	4	14.8	14.8
Demo Only	3	11.1	25.9
CCR Only	5	18.5	44.4
Demo + CCR	15	55.6	100.0
Total	27	100.0	—

The Demonstration + CCR condition was also the only condition in which subjects did *not* stay the majority of the time: 50% chose to stay, and 50% chose switch. The percentage of subjects who switched in the other conditions was much lower. As shown in Table 3, a Pearson Chi-Square analysis revealed the number of subjects who switched in the Demonstration + CCR Task condition to be statistically significant above the other three conditions: χ^2 (3, *N = 96)* = 10.58, *p* = .014, Cramér's *V* = .332. It is also the only condition in which the *observed count* of switches (n_o = 15) actually exceeded the *expected count* (n_e = 7.3), more than doubling the value.

Table 3. *Non-parametric analysis of the decision to 'stay' or 'switch' by condition.*

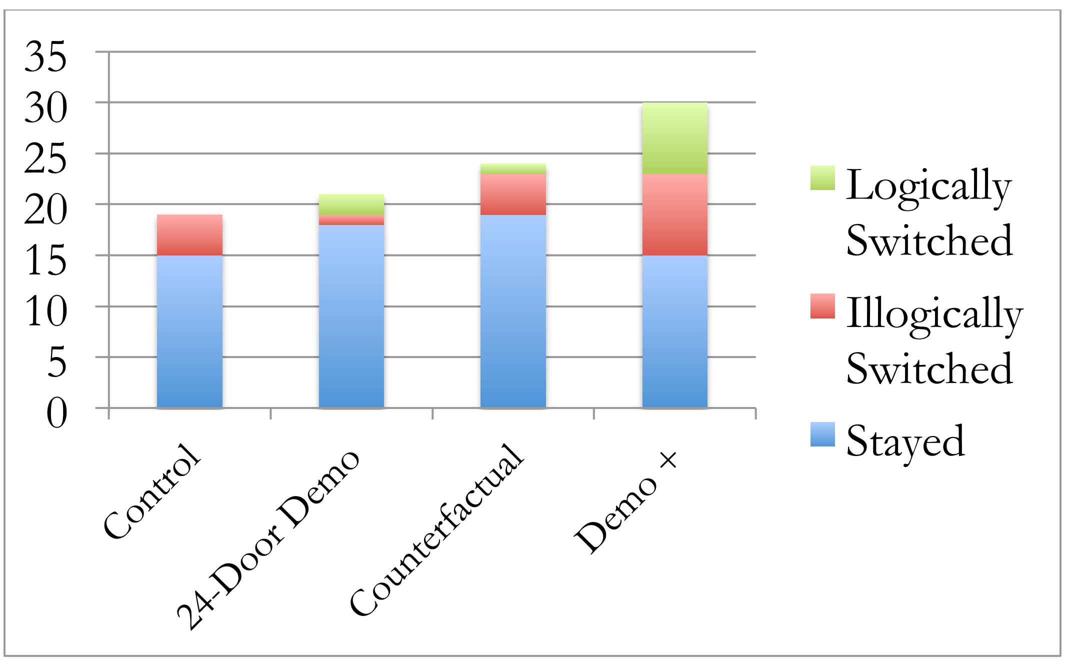
			Decision		
			Stay	Switch	Total
Condition	Control	Count	15	4	19
		Expected Count	13.7	5.3	19.0
	Demo	Count	18	3	21
		Expected Count	15.1	5.9	21.0
	CCR	Count	21	5	26
		Expected Count	18.7	7.3	26.0
	Demo + CCR	Count	15	15	30
		Expected Count	21.6	8.4	30.0
Total		Count	69	27	96
		Expected Count	69.0	27.0	96.0

The self-reported reasons for switching (instead of staying) fell into three categories: people reported wanting to go with their lucky number; people believed the experimenters were playing some sort of trick (labeled 'misinterpretation'); and, most importantly, because they had the correct insight about the probabilistic advantage. Referring to both Figure 4 and Table 4, of the 27 subjects who switched, only *ten* did so because they realized it was the optimal decision. Of those ten subjects, seven were in the Demonstration + CCR Task condition. A Pearson Chi-Square analysis revealed this to be statistically significant: χ^2 (6, 27) = 12.696, *p* = .048, Cramér's *V* = .485. Two of the remaining subjects who switched for the correct reason were in the Control condition, and one was in the CCR Task condition. It must be noted that this means that overall out of 96 subjects, ten correctly reasoned the Monty Hall problem—that is, 10.4%.

Table 4. *Non-parametric analysis of the subjects who decided to 'switch' and the reasons for doing so.*

			Reason for 'Switching'			
			Lucky Number	Misinterpretation	Correct Reason	Total
Condition	Control	Count	1	1	2	4
		Expected Count	1.5	1.0	1.5	4.0
	Demo	Count	0	2	1	3
		Expected Count	1.1	.8	1.1	3.0
	CCR	Count	5	0	0	5
		Expected Count	1.9	1.3	1.9	5.0
	Demo + CCR	Count	4	4	7	15
		Expected Count	5.6	3.9	5.6	15.0
Total		Count	10	7	10	27
		Expected Count	10.0	7.0	10.0	27.0

Figure 4. *Graph depicting the percentages of subjects who chose to 'stay' and 'switch' across the four conditions.*



**Note that 'Logically Switched' represents those subjects who switch for the correct reason.*

Discussion

In general, our hypotheses were borne out. The subjects in the four randomized conditions rarely chose to 'switch' cumulatively (n = 27), and only ten subjects did so because they realized it improved their odds of winning. Of those ten subjects who decided to switch to the other door, seven were in the Demonstration + CCR Task condition. The proportion of subjects switching in the other conditions was not above chance. Given these finding, there appears to be an additive effect when the demonstration condition and counterfactual conditional reasoning tasks were combined.

Based on subjects' written explanations for their decisions to stay, the 28-door demonstration by itself seemed only to make subjects think more creatively about why they would stay. And while it was hypothesized that counterfactual conditional reasoning tasks would encourage more subjects to switch (regardless of the reason), they did not. However, when these two conditions were combined, subjects were statistically significantly more likely to switch. Further, though it was still rare, subjects in the Demonstration + Control Condition sometimes switched because they grasped the underlying logic (n = 7).

Since the 28-door demonstration by itself did not improve subjects' understanding of the Monty Hall problem, we can conclude that, for our study, perspective change was not helpful. The results suggest that the 28-door demonstration seems to have had the effect of freeing up subjects' thinking about the problem, but that it is poorly formulated as a logical proposition. The effect of this is that while subjects reasoned through the 28-door demonstration accurately (i.e., the 52 subjects in the study all said that their friend should *switch*), the majority of these subjects did not then apply the same reasoning to the Monty Hall problem itself, *even though the underlying logical form is the same* for the 28-door demonstration and the three-door Monty Hall problem. The counterfactual conditional reasoning tasks seemed to have helped refine reasoning after the 28-door demonstration, since they embody a rather rigid underlying logical form, and the CCR tasks may help facilitate a logical, propositional attitude toward the problem when combined with the demonstration.

A criticism of our study could be that there might not have been an additive, enhancing benefit that is directly attributable to the combination of the 28-door demonstration and CCR tasks, since it could be that the condition simply required more time or effort. While this is certainly valid, given that seven out of ten subjects who correctly responded to the Monty Hall problem were in the Demonstration + CCR condition, it would seem that something was happening that was beneficial. Further, the benefits of these tasks, or those like it (see Krauss & Wang, 2003; Burns & Weith, 2005) have been established previously. Unfortunately, what this study did not do is replicate those benefits, since neither the 28-door demonstration nor the CCR Tasks alone were of any help to the subjects in this study.

GENERAL DISCUSSION

To widen the scope of the implications of our study, we can say that our findings are in keeping with much of the research on cognitive biases. As many know, this work began with Kahneman and Tversky (1972), and the finding of much of the work over the past decades was summarized and popularized to the general public in Daniel Kahneman's best-selling book *Thinking, Fast and Slow* (2011). The argument is that humans have two systems of thinking: System 1, which is defined by quick, heuristic thinking; and System 2, which is defined by slow, logical thinking. Further, much of the research suggests that humans favor System 1 over System 2 thinking.

Our findings are largely in keeping with Kahneman's argument. In our study, almost no one seemed to understand that with the Monty Hall problem the contestant should always switch. That is, only ten out of 96 people correctly reasoned their way to the conclusion that one should switch because of the probabilities. And, further, this was generally accomplished only after substantial encouragement: seven of the ten subjects who made the realization to switch did so after first seeing a demonstration essentially giving away the answer to the Monty Hall problem and then completing a series of moderately difficult logical tasks. In our study, it is fair to conclude that nearly everyone engaged in System 1 thinking and System 1 thinking only.

Kahneman (2011) has argued that it is only with a fair amount of effort that the vast majority of humans engage in slow, System 2 thinking. Our findings support this argument too. There were ten people who switched from fast thinking to slow thinking, and seven subjects went through both an illustrative demonstration and a series of logical tasks.

We would, however, like to take this one step further. When subjects went through the 28-door demonstration and correctly decided that the best decision was to switch, they were certainly using slow, System 2 thinking. And when subjects completed the counterfactuals they were also using slow thinking. However, in neither of these situations was the slow thinking carried forward in any meaningful way up through the Monty Hall problem. It was only when these two situations were combined that there was any appreciable effect on human thinking. We suggest that not only can human cognition be bifurcated into two systems—fast and slow—but the slow, logical system itself has two subsystems. We'd like to call them 'fast logic' and 'slow logic.'

As Nisbett has argued, not all logical problems and puzzles are created equal (e.g., see Nisbett, Fong, Lehman, & Chang, 1987). Some logical problems are easier, requiring only a 'faster' slow thinking. There are two examples of this in our study: the 28-door demonstration and the counterfactual reasoning tasks. Both require a somewhat slower form of thinking than is typical of faster, heuristic thinking. But the difficulty is moderate. Fifty-one out of 54 subjects understood the 28-door demonstration (three subjects were left out of the analysis from the Demonstration Only condition). Also, there were few errors on the counterfactual reasoning tasks, and most of those had to do with the most challenging type, the double-negative '*not-p* and *not-q*.' However, some logical problems are harder. One classic example is Wason Selection Task (see Wason, 1968; and for a more recent study see Taylor, Ashworth, Petrovich, & Young, 2017). Roughly, only ten percent of subjects get the problem right. Another example of a distinctly hard logical problem is the Monty Hall problem. Here again we find roughly ten percent get the problem right. Of all of those who did get it right, the vast majority only did so after having to reason through two different but complementary logical problems.

In one interpretation of our study, the results strongly support Kahenman's long-held argument that humans are not very good at thinking logically, and they struggle to be encouraged or taught to do so. An additional interpretation is that, while humans are not great at logical thinking, there is some evidence here that they can, with some effort, be encouraged to do so. On one hand, only ten subjects understood the logic of the Monty Hall problem. On the other hand, almost all of them were in the one condition that challenged subjects with two reasoning tasks first. So while only 23 percent of subjects switched under these conditions (28-door demonstration plus the counterfactuals), it was 4.5 percent for the other three conditions combined. That strongly suggests that there was a measurable, meaningful effect.

While there have been countless studies showing the weaknesses of human logical reasoning (like the Wason Selection Task), the Monty Hall problem demonstrates a particular limitation. The solution to the problem is explained through Bayesian logic, and human beings are weak at Bayesian inference and adjusting Bayesian base rates (Baratgin, 2009). And while situations and context can facilitate Bayesian reasoning and the corresponding adjustment of base rates, overall human reasoners are not strong and are still subject to bias (Cohen, Sidlowski, & Staub, 2017).

The implications for evolutionary cognitive psychology are that the human mind has evolved over millennia to default to fast, heuristic thinking while simultaneously having evolved for the *capacity* for slow, logical thinking. Further, humans have evolved to have different degrees or modes of slow thinking, called here 'fast logic' and 'slow logic.' It is this latter capacity which requires the most time, calories, and, most importantly for our discussion, cognitive effort. At the far end of cognitively demanding reasoning in the 'slow logic' realm lies Bayesian logic, and somewhere in this realm lies the Monty Hall problem. From our study and many of the research cited within, one can easily conclude that humans have not evolved to handle Bayesian reasoning or the Monty Hall problem readily. What our study and others also show is a capacity to do so with the proper context, learning, and experience. Human cognition has evolved the capacity for 'slow logic.' It appears, however, that we engage in it only with substantial effort much of the time.

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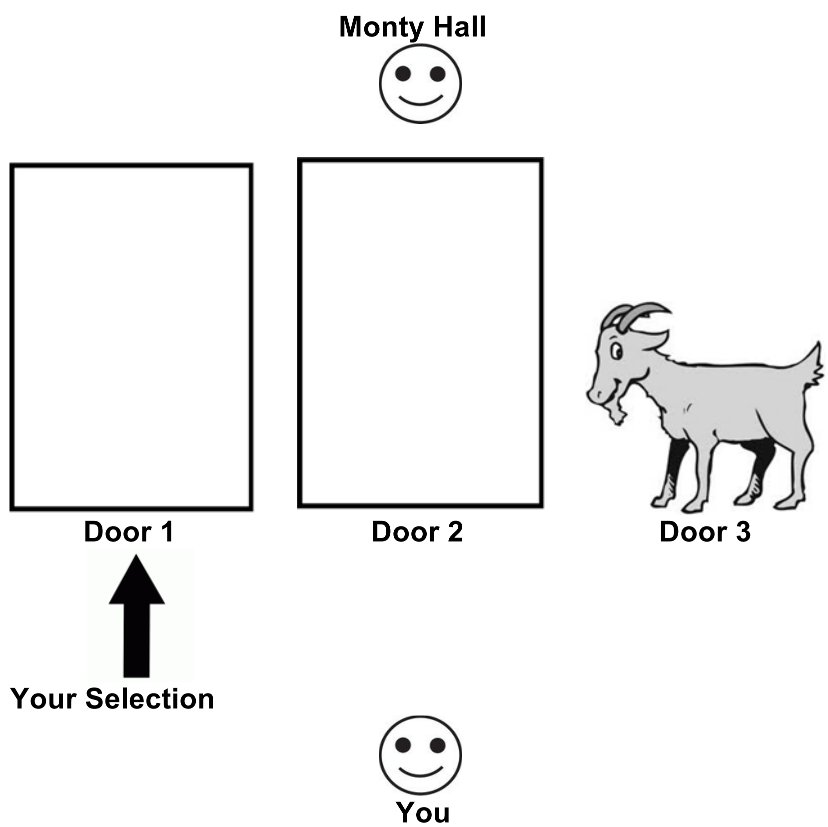
APPENDIX A

LET'S MAKE A DEAL

There is a game called 'Let's Make a Deal,' in which a contestant is allowed to choose one of three closed doors. Behind one closed door is the prize, a new car. Behind each of the other two doors are goats. Imagine you are the contestant. After you have chosen a door, all three doors remain closed for the time being. According to the rules of the game, the game show host (named Monty Hall), who knows what is behind each of the three doors, now has to open one of the two un-chosen doors to reveal a goat. After Monty shows a goat to you, he asks you, the contestant, to decide *whether you want to stay with your first choice or if you would like to switch to the last remaining door.*

Task:

Again, imagine that you are the contestant, and you do not know which of the doors the car is behind. You now randomly choose Door 1.



According to the rules, Monty then reveals a goat behind one of the doors—in this case Door 3. Now he asks whether you want to stay with the first choice (in this case, Door 1) or do you want to switch the other door (in this case, Door 2). What should you do (please select one)?

_____ Stay or _____Switch

Please tell me in writing what went on in your head when you made your decision. Feel free to use sketches, etc. to explain your answer.

Please also tell me if you were already familiar with this game ____ (yes) ____ (no).

Please also tell me if you already know the correct answer ____ (yes) ____ (no).

Thank you very much for your participation!

APPENDIX B

The Counterfactual Conditional Reasoning Tasks (correct responses are in the parentheses).

1. If it had been raining, then the grass would be _____ (wet).
2. If it had *not* been raining, then the grass would *not* be _____ (wet).
3. If it had *not* been raining, then the grass would be _____ (dry).
4. If it had been raining, then the grass would *not* be _____ (dry).
5. If the pencil had been sharpened, then it would be _____ (sharp).
6. If the pencil had *not* been sharpened, then it would *not* be _____ (sharp).
7. If the pencil had *not* been sharpened, then it would be _____ (dull).
8. If the pencil had been sharpened, then it would *not* be _____ (dull).
9. If he had been breathing, then he would be _____ (alive).
10. If he had *not* been breathing, then he would *not* be _____ (alive).
11. If he had *not* been breathing, then he would be _____ (dead).
12. If he had been breathing, then he would *not* be _____ (dead).
13. If the wind had been blowing, the sailboat would be _____ (sailing).
14. If the wind had *not* been blowing, the sailboat would *not* be _____ (sailing).
15. If the wind had *not* been blowing, the sailboat would be _____ (still).
16. If the wind had been blowing, the sailboat would *not* be _____ (still).
17. If she had been wed, then she would be _____ (married).
18. If she had *not* been wed, then she would *not* be _____ (married).
19. If she had *not* been wed, then she would be _____ (single).
20. If she had been wed, then she would *not* be _____ (single).

APPENDIX C

Four key images in the sequence for the Demonstration conditions.

Image 1:

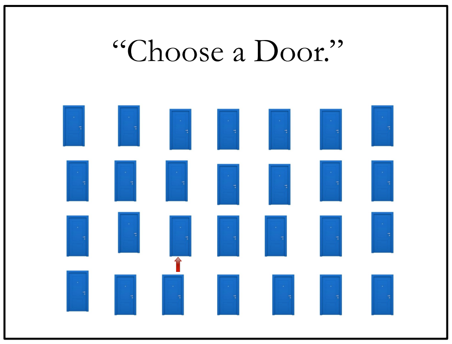


Image 2:

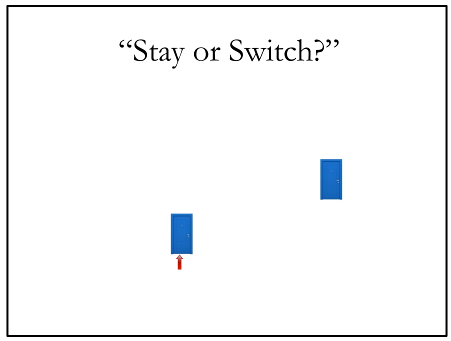


Image 3:

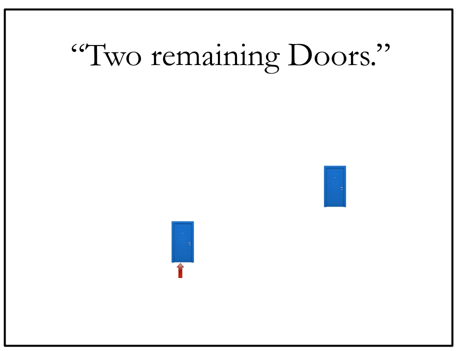


Image 4: